Three's the Charm

Triple-Frequency Combinations in Future GNSS

With the advent of multi-frequency global navigation satellite systems, triple-frequency combinations of measurements shall become available that do not include information on receiver or satellite positions, velocities and clock biases (geometry-free), and no atmospheric delays as well (iono-free). These will offer important applications in future GNSS, in particular in multipath analysis and multi-frequency ambiguity resolution algorithms.

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Geometry-free combinations of measurements from the same satellite, such as ($\Phi_1 - \Phi_2$) or ($P_1 - P_2$), are used in today's two-frequency GNSS to estimate variations of inter-frequency ionosphere delays. These combinations also contain information on multipath, which cannot be separated from ionosphere delays if only two frequencies are used.

If the measurements on a third frequency are available, these dual-frequency geometry-free combinations can be combined together to form triple-frequency geometry-free/iono-free combinations, which contain the superposition of multipath and tracking errors for the three frequencies, while ionosphere delays are canceled out.

These combinations can be formed for both phase and code measurements, but are particularly useful for phase. Firstly, phase multipath can be extracted from single-station raw measurement data; this task cannot be tackled with only two frequencies. Secondly, triple-frequency combinations of phase measurements are equal to corresponding linear combinations of phase ambiguities and, hence, can be used as constraints in ambiguity resolution algorithms.

In practice, this means that only ambiguities for the two frequencies are independent, while ambiguities on the other frequencies can be related to these two through simple linear formulas. The use of these constraints can reduce the number of independent ambiguities and improve the performance of multi-frequency ambiguity resolution algorithms.

Triple-frequency combinations of code measurements can be used in a similar manner to estimate code multipath and provide a relationship between timing group delays. Similarly to the case of phase ambiguities, timing group delays at any third frequency can be computed from raw range measurements based on the group delays for only two frequencies.

Multipath Analysis Today

The analysis of code multipath in dual-frequency GNSS is traditionally based on the formula

$$M_{P1} = P_1 - \Phi_1 + 2\lambda_1^2 \frac{\Phi_2 - \Phi_1}{\lambda_2^2 - \lambda_1^2} \qquad (1)$$

Here M_1 is code multipath on the signal 1, where P_1, Φ_1, λ_1 are respectively pseudorange, phase expressed in units of length, and wavelength of the same signal. Φ_2, λ_2 are phase and wavelength of another signal at another frequency.

In a nutshell, expression (1) is a difference (code-phase) corrected for ionosphere delays. Combinations (code-phase) for individual signals contain a mix of tracking noise, multipath, and

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ionosphere delays. In order to exclude ionosphere delays, a linear combination of two phases must be used. The derivation of (1) relies on the fact that ionosphere delays are in a very good approximation proportional to λ^2, where λ is the wavelength.

Formula (1) has the following advantages:

- it furnishes multipath for only one code
- it requires the data from only one receiver
- it can be used with both static and kinematic data.

The drawback of (1) is that it contains phase ambiguities, which entails two kinds of problems:

- absolute values of multipath are unknown, only variations are available,
- multipath variations can be estimated only over the periods of continuous tracking when phase ambiguities do not change.

The second problem is particularly limiting for the analysis of kinematic data in highly masked environments.

As for the phase multipath, no single-station indicators are available in dual-frequency GNSS. The only known way to extract phase multipath information is through residuals of short baseline processing. This method requires two receiver stations and has significant drawbacks as follows:

- it requires sophisticated processing software
- if double-differencing is used, which is most typical, the residuals are not referred to individual satellites
- if single-differencing is used (which may not be available in standard post-processing SW), the estimate of differential clock error is mixed with residuals
- the multipath errors of the two stations are mixed and could be partly correlated if the stations are too close together.

In the multi-frequency GNSS of the future, a new approach to estimate single-station multipath shall become available.

> With triple-frequency techniques, we can extract phase multipath from single-station raw measurement data and use phase measurements as constraints in ambiguity resolution algorithms.

Triple-Frequency Phase Multipath

Triple-frequency multipath combination is a generalization of a well-known dual-frequency geometry-free combination $\Phi_1 - \Phi_2$, which is widely used to analyze phase noise and detect cycle slips. It contains a superposition of tracking noise, multipath, and inter-frequency ionosphere delays; so, it cannot be directly used in multipath research. However, it is easy to prove that a simple linear combination of three phase differences shall contain no ionosphere delays:

$$M_{\Phi 123} = \lambda_3^2(\Phi_1 - \Phi_2) + \lambda_2^2(\Phi_3 - \Phi_1) + \lambda_1^2(\Phi_2 - \Phi_3) \quad (2)$$

The derivation of (2) is quite simple and is based on the same assumptions as the derivation of (1). The structure of this combination is the same for all the three kinds of measurements: ranges, phases, and Doppler measurements:

$$M_{P123} = \lambda_3^2(P_1 - P_2) + \lambda_2^2(P_3 - P_1) + \lambda_1^2(P_2 - P_3) \quad (3)$$

$$M_{D123} = \lambda_3^2(D_1 - D_2) + \lambda_2^2(D_3 - D_1) + \lambda_1^2(D_2 - D_3) \quad (4)$$

Here D_i are Doppler measurements expressed in the units of linear velocity. We should stress that triple-frequency multipath combinations furnish a weighted sum of multipath/tracking errors for the same satellite on all the three frequencies. The contributions of multipath errors for individual signals depend upon the wavelength factors. Let us emphasize again that the indexing in (2)-(4) corresponds to frequencies, not satellites. The triple-frequency combinations contain measurements from the same satellite but on three different frequencies.

The triple-frequency phase combinations provide a valuable indication of phase multipath from single-station data, which is not available in today's two-frequency GNSS. The limitation of this indicator is that the multipath for individual signals cannot be extracted; only a combination of three multipath errors is provided. In addition to multipath and tracking noise, combinations (2)-(4) are affected by second-order ionosphere delays (usually millimeter-level) and possibly by thermal drift of inter-frequency hardware delays.

We should note that, although it is common knowledge that Galileo transmits in three frequency bands — L1, E5 and E6, in reality five distinct frequencies exist: L1, E5a, E5b, E5AltBOC, and E6. Therefore, a large number of triple-frequency combinations can be made. Even with only open-service signals of L1 and E5, four different combinations shall be available.

Multi-Frequency Ambiguity Resolution

Formula (2) can be extended to include phase ambiguities. Assuming that phase ambiguities are equal to phase values at some initial point in time (beginning of tracking), this formula can be rewritten in the following form:

$$\lambda_3^2(\Phi_1 - \Phi_2) + \lambda_2^2(\Phi_3 - \Phi_1) + \lambda_1^2(\Phi_2 - \Phi_3) = \\ = \lambda_3^2(B_1 - B_2) + \lambda_2^2(B_3 - B_1) + \lambda_1^2(B_2 - B_3) \quad (5)$$

Here B_1, B_2, B_3 are floating ambiguities (also in meters), which include both integer ambiguities and HW delays on both receiver and satellite sides. This equation is not a precise identity: the right hand side is constant, but the left hand side includes multipath and tracking noise of phase measurements.

Formula (5) can be seen from two perspectives. On the one hand, we can focus on the time-dependent variations of the left hand side (in fact, on deviations from (5)), which provide the measure of the phase multipath. This has already been discussed in the previous section.

On the other hand, (5) can be seen as a relationship between ambiguities.

Formula (5) states that phase ambiguities for three phases must conform to a constraint, the value of which can be easily computed from a single set of three phase values for the one epoch. By using phase measurements for many epochs, this constraint may be made more precise through time averaging.

This constraint may be used by any navigation algorithm, which uses floating ambiguities. Navigation algorithms, which use phase observables, usually include phase ambiguities as unknown constant values (Kalman filter states), which are to be determined in the processing in addition to position and velocity components. Any known relationship between ambiguities can be used as a constraint to increase the redundancy of ambiguity resolution.

Differential forms of equation (5), same in appearance as equation (5) itself, can be introduced for differential carrier-phase processing. For the purposes of integer ambiguity resolution, let us consider a double-differenced form of equation (5). This form can be easily derived from (5) based on the well-known fact that all the non-integer ambiguity components disappear in double-differencing:

$$\lambda_3^2(\Delta\nabla\Phi_1-\Delta\nabla\Phi_2)+\lambda_2^2(\Delta\nabla\Phi_3-\Delta\nabla\Phi_1)+\lambda_1^2(\Delta\nabla\Phi_2-\Delta\nabla\Phi_3)= \\ =\lambda_3^2(\lambda_1\Delta\nabla N_1-\lambda_2\Delta\nabla N_2)+\lambda_2^2(\lambda_3\Delta\nabla N_3-\lambda_1\Delta\nabla N_1)+\lambda_1^2(\lambda_2\Delta\nabla N_2-\lambda_3\Delta\nabla N_3) \quad (6)$$

Here N_1, N_2, N_3 are integer ambiguities for three frequencies for a certain pair of stations/satellites used in double differencing. Integer ambiguities are related to floating-point ambiguities through a relationship $B_i = \lambda_i N_i + (HW\ biases)_i$, where hardware biases disappear in double differencing. Just to reiterate, in equations (1),(2),(5),(6) phase is measured in units of length (meters).

Equation (6) can be used as a constraint by any multi-frequency integer ambiguity resolution algorithm for double-differencing differential carrier phase processing. With this constraint, the efficiency of the ambiguity resolution shall increase. In fact, with the introduction of the third phase no new unknowns are introduced: ambiguities on any third frequency shall be computed if the ambiguities on the two frequencies are known.

If the tracking of one phase out of three is lost and recovered, the new ambiguity can be immediately recomputed from equation (6) without involving the navigation processing.

In particular, this means that if the tracking of one phase out of three is lost and recovered, the new ambiguity can be immediately recomputed from equation (6) without involving the navigation processing. If the number of involved frequencies increases, ambiguities must be resolved only for two frequencies, all the other ambiguities can be seen as dependent. Future algorithms are likely to use symmetrical approach: handle all the ambiguities as unknowns and use relationships (6) as constraints (for example, introduce them as very precise quasi-measurements).

Multi-System Ambiguity Resolution

Multi-system multi-frequency ambiguity resolution has already attracted significant attention. Two major approaches exist:

- Combining of the two systems, where only double differences between the satellites of the same system are used (For further details, see the paper by S. Verhagen et al listed in the "Additional Resources" section at the end of this article.)
- Cross-coupling approach where the double differences between satellites of different systems are also included. (See the two papers by Julien et al listed in the "Additional Resources" section.)

Equation (5) relates the values that refer to the same satellite and is system-independent: it holds for any satellite, which transmits at least on three frequencies. Equation (6) holds for the measurements and ambiguities of any pair of satellites, which transmit on the same trio of frequencies. Although equation (6) is also in principle system-independent, the two systems can be combined only if they use the same trio of frequencies.

When constraints (6) are applied to the mixed GPS/Galileo processing, we have to take into account that GPS and Galileo have no common trios of frequencies. In practice, this means that with the system-combining approach described by S. Verhagen et al ambiguity constraints (6) can be used directly for all the pairs of satellites, while with the cross-coupling approach described by O. Julien et al they can be used only for the pairs of satellites that belong to the same system.

A large number of unknown ambiguities in multi-frequency, multi-system ambiguity resolution entails a significant computational burden. The number of candidate ambiguity sets increases dramatically. In the May 2004 paper by O. Julien et al, the number of searched candidates was even forcibly limited in order to make the process manageable.

The use of triple-frequency constraints (6) will help to limit the complexity of the search process.

We should expect that the number of ambiguity candidates presented for validation shall not significantly increase compared to the case of two frequencies.

Triple-Frequency Code Multipath

An analog of formula (5) also exists for ranges:

$$\lambda_3^2(P_1-P_2)+\lambda_2^2(P_3-P_1)+\lambda_1^2(P_2-P_3)=\lambda_3^2(c_1-c_2)+\lambda_2^2(c_3-c_1)+\lambda_1^2(c_2-c_3) \quad (7)$$

The right hand side of (7) is a linear combination of code biases; c_i are sums of receiver-side and the satellite-side biases. Similarly to (5), this equation is not a precise identity: the right hand side is a constant, but the left hand side includes code multipath and tracking errors.

As a source of information about code multipath, expression (7) has only a limited value, because it indicates a mix of multipath for all three frequencies, while the classical approach based on equation (1) results in a multipath estimate for only one code range. However, the drawback of equation (1) is that it includes phase ambiguities and hence can indicate only variations of multipath errors within intervals of continuous phase tracking. Each time the tracking is interrupted, a new value of phase ambiguity gives a new unknown bias to the "multipath combination" (1).

The advantage of (7) for multipath assessment is that it does provide an absolute measure of code range multipath errors. This is, in fact, quite a unique quality of equation (7) and particularly valuable for multipath assessment of kinematic data with frequent satellite outages and losses-of-lock. In this case, the estimation given by (1) shall consist of a patchwork of disconnected pieces with different ambiguities, while formula (7) shall produce a series of consistent multipath estimates with the constant ambiguity given by the right hand side.

As with the triple-frequency combination of phase, equation (7) can also be used as a relationship between code biases, the components of the right hand side. So far straightforward code differences at the same frequency, such as C/A –P1 and C2-P2, are typically used as a source of information on code biases. (For further details see the paper by Simsky and Sleewaegen listed in Additional Resources.) Because code biases have the same order of magnitude as multipath errors, they are extracted by long-term averaging of code differences.

Similarly, equation (7) can be used to compute an average estimate of the right hand side, which shall result in a relationship between code biases at different frequencies. In practice, it means that if timing group delays are known for two frequencies, they can be simply computed for all the other frequencies using equation (7). If the receiver is calibrated (code group delays known), these estimates of code biases shall have correct absolute values.

Closing Notes

It is well-known that the second frequency was introduced in GNSS so as to obtain real-time estimates of ionosphere delays. In an imaginary GNSS with no ionospheric effects on signal propagation, the second frequency would not provide any essential new information: it would only be useful as a source of additional data with independent multipath and tracking errors.

In ionosphere-free GNSS, the geometry-free combination would be constant and the relationship between ambiguities on the two frequencies becomes trivial as per (8).

$$(\Delta\nabla\Phi_1 - \Delta\nabla\Phi_2) = (\lambda_1 \Delta\nabla N_1 - \lambda_2 \Delta\nabla N_2) \quad (8)$$

The role of the third frequency in the future GNSS is comparable to that of the second frequency in an imaginary ionosphere-free GNSS. Because two frequencies are sufficient to estimate ionospheric errors, the third frequency does not add any new geometric information; it provides only an independent set of multipath and tracking errors.

Therefore, not surprisingly the measurements on the additional frequencies relate simply to the measurement on the two "basic" frequencies and include neither geometric nor ionospheric information. However, redundancy of observations, which comes with the use of additional frequencies, shall prove quite useful, especially combined with low-multipath characteristics of future Galileo signals.

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Conclusions

In multi-frequency GNSS of the future triple-frequency, geometry-free/iono-free linear combinations of ranging measurements from the same satellite shall become available. These linear combinations contain superposition of multipath and tracking errors on the three frequencies; so, information on phase multipath shall become available through single-satellite single-station measurements. For code multipath this method also presents some advantages in addition to known techniques.

We have also seen that triple-frequency combinations of phase measurements can be translated into linear relationships between phase ambiguities, which can be used as constraints in ambiguity resolution algorithms. Ambiguities on any third frequency can be computed through these relationships if the ambiguities on the two frequencies are known. This will lead to simplification and performance improvement of existing multi-frequency ambiguity resolution schemes. Similar relationships also exist between timing group delays for any trio of frequencies.

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Additional Resources

[1] Julien O., M. E. Cannon, P. Alves, G. Lachapelle, "Triple frequency ambiguity resolution using GPS/GALILEO", European Journal of Navigation, Vol. 2, No. 2, May 2004.

[2] Julien O., P. Alves, M. E. Cannon, W. Zhang, "A tightly coupled GPS/GALILEO combination for improved ambiguity resolution", Proceedings of ENC-GNSS 2003, Graz, Austria, 22-25 April 2003.

[3] Simsky, A. and J.-M. Sleewaegen, "C/A code bias in high-end receivers". Proceedings of ENC-GNSS 2004, Rotterdam, 16-19 May 2004.

[4] Verhagen S. and P. Joosten, "Algorithms for design computations for integrated GPS–GALILEO", Proceedings of ENC-GNSS 2003, Graz, Austria, 22-25 April 2003. IG